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Three-dimensional numerical simulations of slump tests

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ABSTRACT

Results of numerical simulations for two conical geometries, the ASTM cone and the mini-cone, are presented. These results are compared to experimental results in the case of mini-cone and cement pastes. The correlation between slump and yield stress obtained numerically is compared to the correlations obtained by different concrete rheometers.

INTRODUCTION

Stoppage tests in civil engineering consists in measuring the shape of a fresh material deposit after flow occurred… In situ, these fast, cheap and simple tests are preferred to conventional rheological tests (Couette Viscometer for cement pastes, Shaugnessy[1], parallel plates rheometer, Nehdy[2] or specific ones (BTRheom, De Larrard[3] or BML[4] for concrete). These rheometers allows the measurement of the two independent parameters that are needed to describe the rheological behaviour of fresh cementitious materials: the yield stress τ_0 and the plastic viscosity η if their fresh behaviour is approximated by using a Bingham model, Tatersall[5].

Stoppage tests are carried out according to the following general procedure : a mould of a given conical shape is filled with the tested fluid. The mould is lifted and flow occurs. If the shear stress in the tested sample becomes smaller than the yield stress (Schowalter[6]) (i.e. the plasticity criterion is not fulfilled any more), the flow stops. The shape at stoppage is directly linked to the rheological behaviour of the tested material and to the cone geometry and test procedure. On a practical point of view, two geometrical quantities can be measured as the results of the test, the slump and the spread. The slump is the difference between the height of the mould and the height of the tested volume after flow stops. The spread is the final diameter of the collapsed sample.

RELATIONS BETWEEN FRESH BEHAVIOUR AND SHAPE AT STOPPAGE

Several attempts can be found in the literature in order to relate slump to yield stress. Schowalter[6] and Murata[7] wrote a relation between a dimensionless slump and a dimensionless yield stress by assuming that the cone could be divided into two parts. In the upper part, the shear stress did not reach the yield stress and no flow occurred. In the lower part of the cone, the shear stress induced by the tested material own weight was higher than the yield stress and flow occurred. The height of the flowing lower part was decreasing until the shear stress in this zone was equal to the yield stress and flow stopped. Schowalter[6] wrote a relation between the final total height of the cone and the yield stress that did not depend on the mould geometry. This relation or similar ones were successfully validated by Clayton[8] or Saak[9] in the case of cylindrical moulds. However, in the case of conical moulds, a discrepancy between predicted and measured slumps was systematically obtained (see Results from Pashias[10], Fig. 1). In the above studies, the experimental results suggested that these relations and the fact that they did not depend on the mould geometry seemed more valid for high slumps (i.e. low yield stress) (Clayton[8], Pashias[9], Saak[10]).

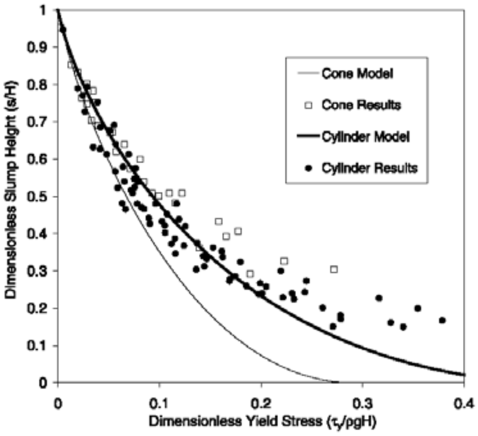


Figure 1 : comparison between predicted and measured slump for the cylindrical and conical geometries. In the case of conical moulds, the predicted slump by is systematically lower than the measured slump (from Clayton et al.8).

For very high slumps, it becomes handier to measure spread instead of slump although both values have been linked together by Coussot[11] or Domone[12]. Coussot[11] wrote a solution of this purely shearing flow valid in the case of high slumps/large spreads. In this regime, the depth of the fluid becomes small in front of the characteristics length of the base of the deposit.

It should be noted that all the above approaches involve a mono-dimensional plasticity criterion and behaviour law. Flow occurs or stops when the shear stress becomes higher or lower than the yield stress. This simplification eases the analysis of the flow but is valid only if the flow is dominated by shear stresses and if the diagonal terms of the deviatoric stress tensor can be neglected compared to the shear stress. In fact, this assumption is true only in the ideal two-dimensional case studied by Coussot[11] (high slumps). When the shape is still conical, the flow is far from being purely shearing and the deviatoric stresses should be taken into account in proper three dimensional flow criterion, Lipscomb[13].

Several authors have also developed numerical simulations of this free surface stoppage flow. Tanigawa[14] developed a visco-plastic finite element analysis introducing a frictional interface law at the base of the slumping cone. He calculated the slump in terms of the yield stress but, as he did not have any experimental way to measure the rheological parameters of concrete, he did not compare his results to experimental measurements. Later, Schowalter[6] compared his analytical prediction to Tanigawa numerical results and found a good agreement. It should be noted that both predictions were based on a mono dimensional plasticity criterion. Hu[15] assumed that the shape of the deposit stayed conical and calculated the state of stress using an elastoplastic finite element analysis. Once again, a mono-dimensional plasticity criterion was considered.

Apart from Tanigawa[14], most analysis are carried out assuming a sticky flow at the base of the deposit. In the case of fluid concretes (low yield stress, high slumps), this assumption is licit as these concretes behaves just like suspensions but in the case of high yield stress, this assumption may be discussed. A concrete with an high yield stress may be obtained by two different trends in the mix fitting. On one hand, the amount of cement or fine particles may be high. The colloidal forces network that can be built between these fine particles increases the yield stress of the mixture. The concrete is similar to a dense fine suspension and, in this case, the assumption of a sticking flow is also licit as experimentally obtained by Pashias[10]. On the other hand, the amount of coarse particles may be high. The behaviour of the obtained concrete becomes closer to the behaviour of a cohesive granular material. The apparent yield stress of such a mixture is similar to the plastic yield value studied in the soil mechanics field. In this case, as for a granular material, the behaviour at the interface may be frictional. This uncertainty in the behaviour at the interface for high yield stress concrete has the following consequence on the analysis any theoretical results obtained with a sticky flow assumption: the results obtained high dimensionless yield stress should be considered with care as the validity of the assumption of a sticking flow depends on the tested material aspect and mix fitting.

The aim of the first part of the present work is to present results of numerical simulations using three dimensional writing of the behaviour law and plasticity criterion for two classical conical geometries, the ASTM[16] cone and the mini cone test. The first one is the standard cone geometry for concrete testing while the second one is used to test cement pastes or mortars. The geometries of these cones are given in Fig.2 and Tab.1.

As disagreement still exists in the field of concrete rheometers (Ferraris[17]), it is difficult to check the validity of any relation (analytical or numerical) between slump and yield stress in the case of ASTM cone and concrete. If concrete is replaced in the ASTM cone with cement pastes or any fine particles suspensions that can be studied using standard viscometry, their plastic viscosity is so low compared to plastic viscosity of concrete that inertia effects can not be neglected. The speed at which the mould is lifted becomes most of the time one of the main parameters of the test.

That is why, in this study, the numerical results are first compared to mini cone test results and standard Vane test measurements on cement pastes. The relatively flat shape of the Mini Cone reduces inertia effects and yield stress is fairly easy to measure in the case of cement pastes, Nguyen[18]. The good agreement between the obtained numerical results and experimental values then allows us to use the numerical results obtained for ASTM cone to predict slump in terms of yield stress.

In the second part, the numerical relation between slump and yield stress is compared to the correlation between slump and yield stress obtained for three concrete rheometers, the BTRheom, the BML and the two points test.

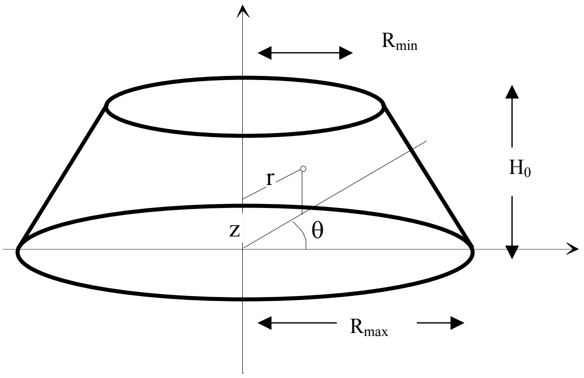


Figure 2. Initial cone shape

Table 1 : cone geometries

Cone	ASTM cone	Mini-cone
H_0 (mm)	300	50
R_{min}(mm)	200	35
R_{max} (mm)	100	50

NUMERICAL SIMULATIONS

The axial symmetry is assumed (no angular velocity). The cylindrical frame of reference (O, r, θ, z) is shown on Fig. 2. p is the pressure, σ_{ij} is the stress tensor and $\sigma_{ij}^{(d)}$ is the deviatoric stress tensor. The chosen plastic criterion is the von Mises three-dimensional yield condition. ρ is the tested material density. s is the slump. s' is the dimensionless slump and τ_0' the dimensionless yield stress as defined by Showalter[6].

$$s' = s/H_0$$

$$\tau_0' = \tau_0/\rho g\, H_0$$

The computational fluid mechanics code Flow3D® is used in this study. An elasto-visco-plastic model is chosen to describe the tested fluid behaviour. The material behaves as an incompressible elastic solid up to the yield stress, beyond which it behaves as a

Bingham fluid. A pictorial view of such a model is shown in Fig. 3. If a stress is imposed on the pictured system, the spring would support the vast majority of the stress, and the behavior would be elastic. As the tensile stress rises, the yield stress limit is exceeded and slip occurs to maintain the elastic stress within the spring at a constant value. Any additional stress is due to viscosity. The divergence of the elastic stress is added to the momentum balance. As the elastic stress model is implemented explicitly, a stability criterion exists to limit the time step as necessary to achieve a numerically stable solution.

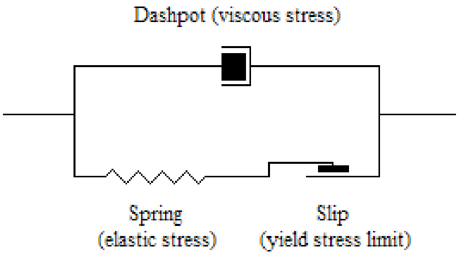


Figure 3. Pictorial view of elasto-visco-plastic stress model, showing the relationship between the elastic and viscous stresses.

The invariant generalization of a Bingham fluid used here is the one proposed by Oldroyd[19] based on the von Mises yield criterion:

$$\sigma_{\mathbf{ij}}^{(\mathbf{d})} = \left[\tau_0 / \left| (\frac{1}{2} \mathbf{d} : \mathbf{d})^{1/2} \right| + \eta \right] \mathbf{d} \, , \frac{1}{2} \sigma_{\mathbf{ij}}^{(\mathbf{d})} : \sigma_{\mathbf{ij}}^{(\mathbf{d})} \geq {\tau_0}^2$$

A value of Young modulus had to be chosen. This value had to be as small as possible as an high value of the Young modulus imposes very small time steps. The final chosen value is 10 Mpa. Above this value, the Young modulus does not influence the results of the simulation as viscous effects dominate the studied phenomenon. Below this value, the elastic deformations affect the result of the numerical simulations.

In all the studies presented in §1, inertia effects are neglected. Because of this simplifying assumption, the influence of the lifting speed of the mould (that depends on the operator) or of the plastic viscosity on the measured slump have not been studied. In other words, the velocity of the flow and its mean kinetic energy are not taken into account and the final shape is calculated as a quasi static asymptotic state assuming it is reached slowly enough. If the plastic viscosity is high, the flow is rather slow and the inertia effects can indeed be neglected. However, if the plastic viscosity is low, the material may keep on flowing because of its kinetic energy and overpass its theoretical stoppage position. Tatersall and Banfill[5] concluded that the slump of fresh concrete is indeed highly correlated with yield stress but is not significantly affected by the plastic viscosity. This conclusion was also reached by Murata[7]. It can be noted that their experiments were carried out on concretes with plastic viscosities rather high compared to the plastic viscosities of contemporary concretes. The plastic viscosity of traditional concrete is rather high (>100 Pa.s) but modern concrete displays plastic viscosity lower than 100 Pa.s. The most fluid concretes, known as self-compacting concretes, display plastic viscosities lower than 40 Pa.s, Wallevik[20]. Only the final state (the shape at stoppage) is studied here and the dimensionless yield tress/dimensionless slump relation is our final objective. However, a value of the plastic viscosity has to be chosen to carry out the simulations. In the present work, the speed at which the mould is lifted is infinite as the mould simply disappears at t = 0 s. In order to get rid of any inertia effects to simulate results of a mould lifted at a reasonable speed (around 100 mm/s, Tanigawa[14]) for a not too fluid concrete, the calculations are carried out with a plastic viscosity equal to 200 Pa.s for the ASTM cone and 1 Pa.s for the mini-cone to ensure that the effects of inertia taken into account by the code do not affect the numerical results. These high values of the plastic viscosity generate "slow" flows and it can be noted that above 150 Pa.s, the plastic viscosity does not have indeed any influence on the predicted slump. Below 150 Pa.s, this influence depends on the yield stress. This rather complex problem will be the subject of an other publication.

The obtained numerical results are plotted on Fig. 4 for the two cone geometries. The predicted values of the slump confirm the fact that slump depends of course on yield tress and density but also on the tested volume and initial height. The scaling suggested by Schowalter[6], although suitable for his own experimental results, does not apply here apart for high dimensionless yield stresses.

Examples of obtained slumped cones are given on Fig. 5(a) and 5(b) for the ASTM cone.

COMPARISON WITH EXPERIMENTAL RESULTS

Measurements of spread and slumps were carried out on cement pastes using the mini cone geometry given in Tab. 1. The yield stress was measured using a standard Vane test procedure Nguyen[18] . Various mix fittings and additives were used. There was no resting time before lifting of the mould in order to prevent any flocculation effects due to thixotropy from influencing the measured slump or spread. The measured yield stresses varied from 1-2 Pa to 300-400 Pa. These experimental results are plotted on Fig. 4.

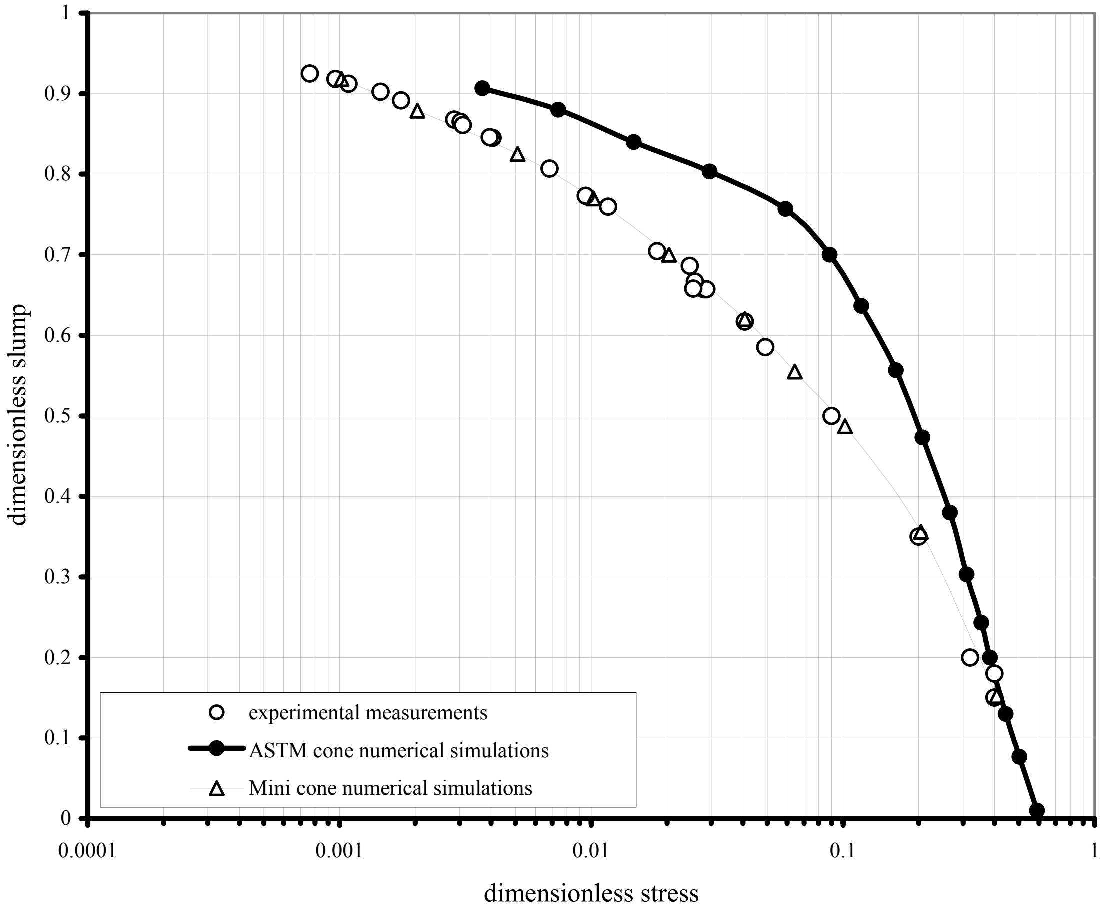


Figure 4: dimensionless slump in terms of dimensionless yield stress for ASTM cone and mini cone. Both numerical predictions and experimental results are plotted for the mini cone test.

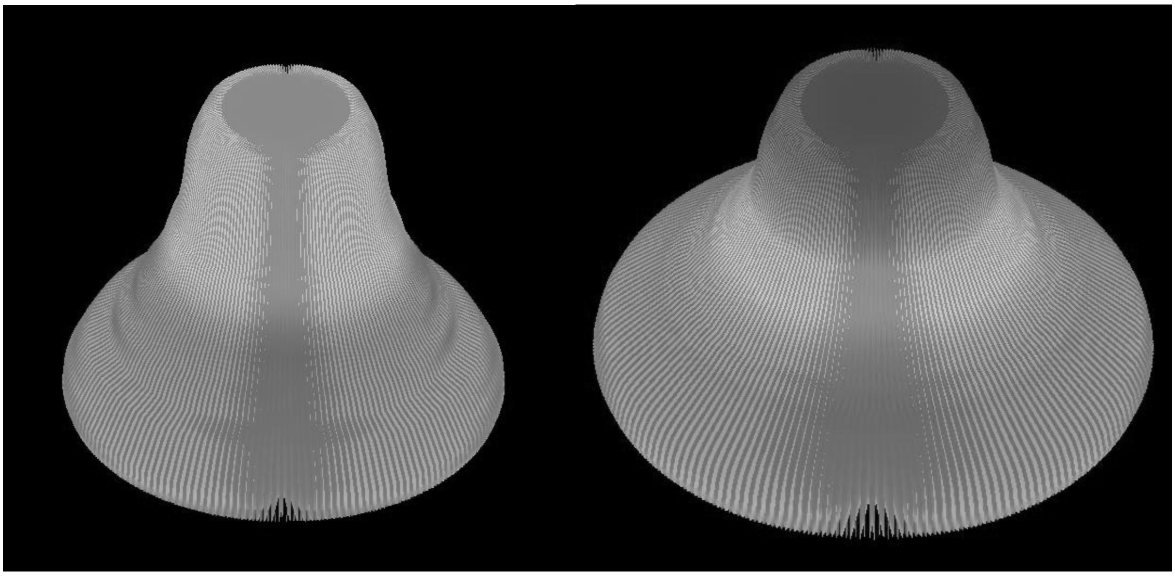

(a) (b)

Figure 5 : Examples of obtained shapes for the ASTM cone (a) yield stress = 2600 Pa (b) yield stress = 2000 Pa.

The good agreement between numerical and experimental results confirm the fact that no sliding occurs at the base of the deposit and that the use of a proper three-dimensional plastic criterion allows correct numerical prediction of the slump or spread.

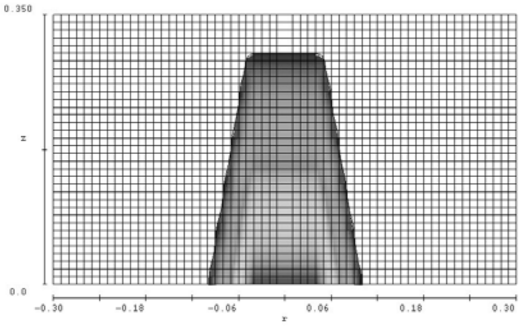


Figure 6. Calculation grid and initial hydrostatic pressure in the concrete before lifting of the mould.

ASTM CONE AND CONCRETE RHEOMETERS

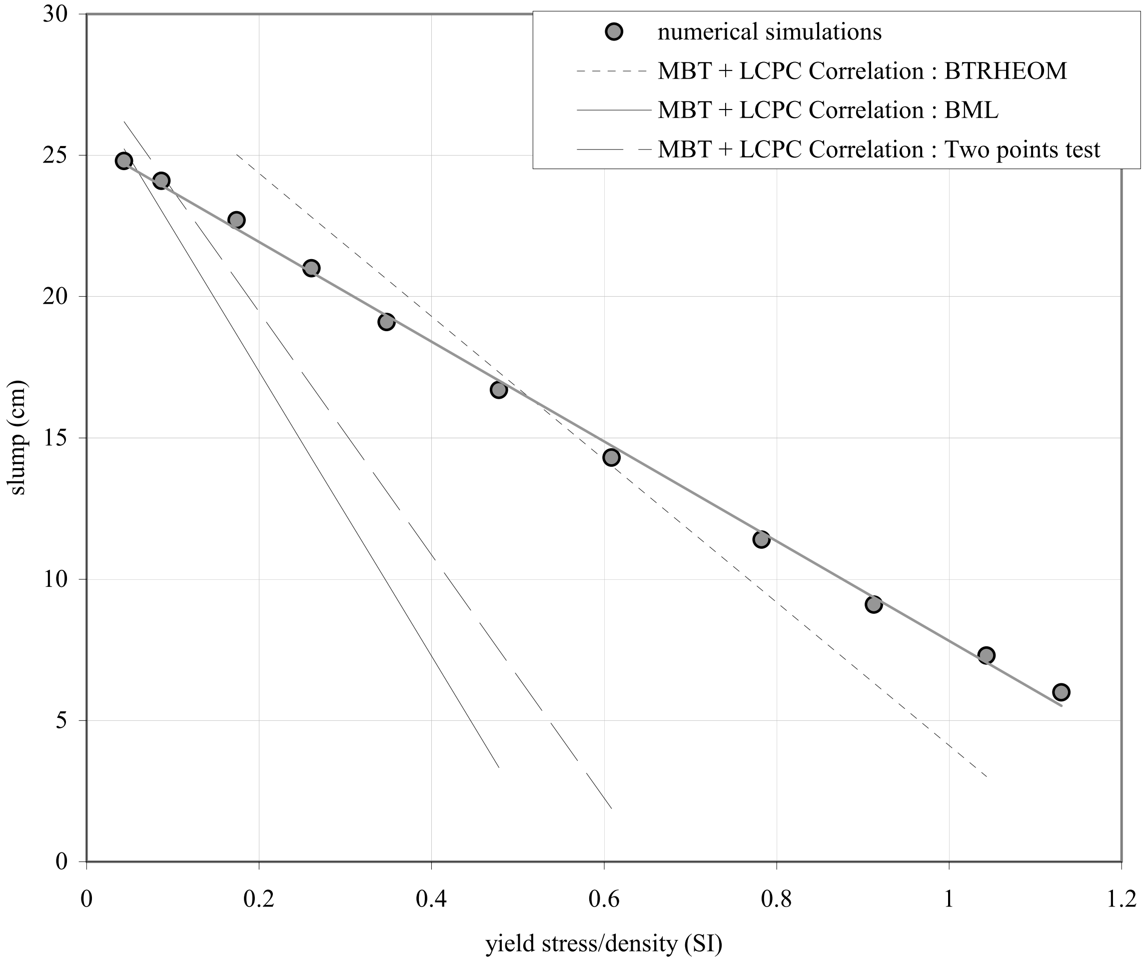
The validation of the proposed numerical method obtained in the previous section allowed us to use the proposed numerical method to predict the ASTM cone slump in terms of the tested concrete yield stress. Apart from the sliding (see §1) that may occur in the case of high yield stress concrete, the flow in the case of the ASTM cone is identical. The numerical correlation between slump in cm and the ratio yield stress/density is plotted on Fig. 7 along with the correlation obtained in the MBT-LCPC comparison campaign between concrete rheometers. The report of this campaign is about to be published. Each correlation curve plotted is the results of 40 measurements on different concretes.

As the behaviour at the base of the deposit is unknown for high yield stresses (possible sliding), the validity of the obtained numerical results should be limited to slumps higher than 5 cm. Moreover, for slumps higher than 25 cm, the depth of the flowing layer of concrete becomes of the same order than the size of the biggest aggregate and the homogeneous fluid mechanics approach proposed here is not valid any more.

However, from these numerical predicted results, a simple linear approximation may be written for slumps between 5 cm and 25 cm.

$$\mathbf{s} = 25.5 - 17.6\frac{\tau_0}{\rho}$$

It is not the aim of the present work to conclude about the efficiency of any concrete rheometer compared to another. However, it seems that, in the 5-20 cm



Yield stress-slump correlations. Both numerical correlations and experimental correlations given by concrete rheometers are plotted.

slump range, the BTRheom correlation between measured yield stress and measured slump is the closest to the numerical correlation. For lower yield stress (self compacting concretes) and higher slump, it seems that the BTRheom correlation overestimates the yield stress. This may be linked to the fact that the sliding assumption needed in the analysis of the BTRheom data is correct in the high yield stress concrete range but, when the yield stress becomes lower, the behaviour at the interface becomes sticky and the shear stress at the interface may not be neglected any more compared to the measured yield stress. The spontaneous formation of a limit layer (made up with water and fine elements) limiting the friction may not occur for low yield stress concrete.

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